EXAM VI
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. What is the x-coordinate of the point of inflection on the graph of $y = xe^x$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

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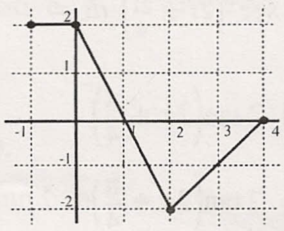
2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown in the figure. If the function H is defined by

$$H(x) = \int_{-1}^{x} f(t)\,dt, \text{ for } -1 \le x \le 4, \text{ then } H(4) =$$

Graph of f



(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

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3. $\int_0^2 |x-1|\,dx =$

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) 2

(E) 3

Ans

4. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x\to c} f(x)$ exists

(B) $\lim_{x\to c} f(x) = f(c)$

(C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

(D) $f(c)$ is defined

(E) $f'(c)$ exists

Ans

5. $\int_0^x 2\sec^2\left(2t+\frac{\pi}{4}\right) dt =$

(A) $2\tan\left(2x+\frac{\pi}{4}\right)$

(B) $2\tan\left(2x+\frac{\pi}{4}\right)-2$

(C) $\tan\left(2x+\frac{\pi}{4}\right)-1$

(D) $2\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$

(E) $\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$

Ans

6. If $xy + x^2 = 6$, then the value of $\dfrac{dy}{dx}$ at $x = -1$ is

(A) -7 (B) -2 (C) 0 (D) 1 (E) 3

Ans

7. $\displaystyle\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{1}{2}$ (C) $\frac{1}{2}\ln 2$ (D) $2\ln 2$ (E) $\ln 2$

Ans

8. Suppose the function *f* is defined so that $f(0) = 1$ and its derivative, f', is given by $f'(x) = e^{\sin x}$. Which of the following statement are TRUE?

I $f''(0) = 1$

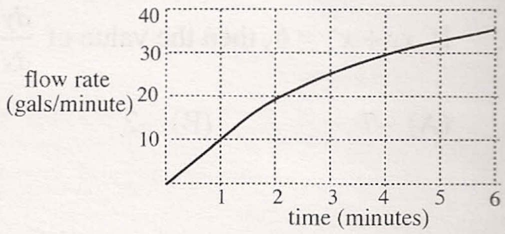
II The line $y = x + 1$ is tangent to the graph of *f* at $x = 0$.

III If $h(x) = f(x^3 - 1)$, then *h* is increasing for all real numbers *x*.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

9. Water flows into a tank at a rate shown in the figure. Of the following, which best approximates the total number of gallons in the tank after 6 minutes?



(A) 75 (B) 95 (C) 115 (D) 135 (E) 155

Ans

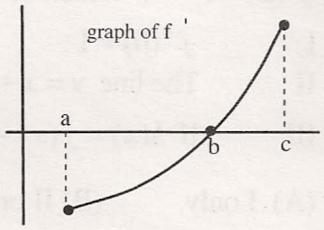
10. What is the instantaneous rate of change at $x = 0$ of the function f given by $f(x) = e^{2x} - 3\sin x$?

(A) –2 (B) –1 (C) 0 (D) 4 (E) 5

Ans

11. Suppose f is a function with continuous first and second derivatives on the closed interval [a, c]. If the graph of its derivative f' is given in the figure, which of the following is true?

(A) f is increasing on the interval (a, b)
(B) f has a relative maximum at $x = b$.
(C) f has an inflection point at $x = b$.
(D) The graph of f is concave down on the interval (a, b).
(E) $\int_a^c f'(x)\,dx = f(c) - f(a)$



Ans

12. Suppose $F(x)=\int_0^{x^2}\frac{1}{2+t^3}\,dt$ for all real x, then $F'(-1)=$

(A) 2 (B) 1 (C) $\frac{1}{3}$ (D) -2 (E) $-\frac{2}{3}$

Ans

13. The graph of the function f is shown in the figure. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only

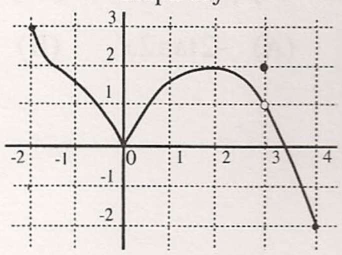
(B) 0 and 2 only

(C) 2 and 3 only

(D) 0 and 3 only

(E) 0, 1 and 3 only

Graph of f



Ans

14. A particle moves along the x-axis so that its position at any time $t\geq 0$ is given by $x(t)=\frac{t}{t^2+4}$. The particle is at rest when $t=$

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) 2 (E) 4

Ans

15. Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval [0,2].

(A) −3

(B) 2

(C) 4

(D) 8

(E) 24

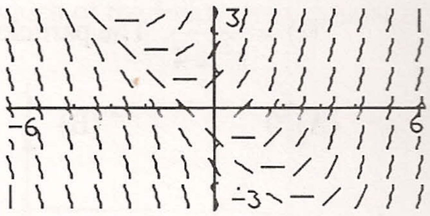
Ans

16. If $f(x) = \ln(\cos 2x)$, then $f'(x) =$

(A) $-2\tan 2x$ (B) $\cot 2x$ (C) $\tan 2x$ (D) $-2\cot 2x$ (E) $2\tan 2x$

Ans

17. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = -y$

(C) $\frac{dy}{dx} = y - \frac{1}{2}y^2$

(D) $\frac{dy}{dx} = x^2 + y^2$

(E) $\frac{dy}{dx} = y^2$

Ans

18. The y-intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point (1, 2) is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) $\frac{7}{4}$

Ans

19. The function defined by $f(x) = (x-1)(x+2)^2$ has inflection points at $x =$

(A) –2 only

(B) –1 only

(C) 0 only

(D) –2 and 0 only

(E) –2 and 1 only

Ans

20. If $\int_0^b (4bx - 2x^2)\,dx = 36$, then $b =$

(A) –6

(B) –3

(C) 3

(D) 6

(E) 15

Ans

21. If $\frac{dy}{dx} = -10y$ and if $y = 50$ when $x = 0$, then $y =$

(A) $50e^{x}$

(B) $50e^{10x}$

(C) $50e^{-10x}$

(D) $50 - 10x$

(E) $50 - 5x^2$

Ans

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22. If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $\left(0, \frac{1}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{5}{3}, 3\right)$ (E) $(3, \infty)$

Ans

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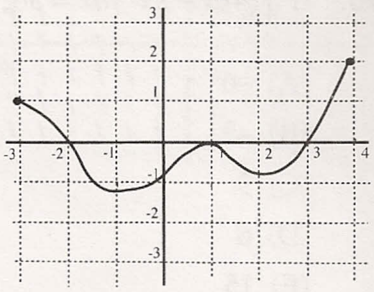
23. The figure shows the graph of f', the derivative of a function f. The domain of f is the closed interval $[-3,4]$. Which of the following is true?

I. f is increasing on the interval (2,4).

II. f has a relative minimum at $x = -2$.

III. The f-graph has an inflection point at $x = 1$.

Graph of f'



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Ans

24. How many critical values does the function $f(x) = \arctan(2x - x^2)$ have?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Ans $\square$

25. Which of the following is continuous at $x = 1$?

I. $f(x) = |x - 1|$

II. $f(x) = e^{x-1}$

III. $f(x) = \ln(e^{x-1} - 1)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

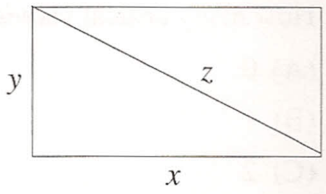
Ans $\square$

26. The number of motels per mile along a 5 mile stretch of highway approaching a city is modeled by the function $m(x) = 11 - e^{0.2x}$, where x is the distance from the city in miles. The approximate number of motels along that stretch of highway is

(A) 16
(B) 26
(C) 36
(D) 46
(E) 56

Ans $\square$

27. The diagonal z of the rectangle at the right is increasing at the rate of 2 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec

(B) $\frac{3}{4}$ cm/sec

(C) $\frac{2}{3}$ cm/sec

(D) $\frac{1}{3}$ cm/sec

(E) $\frac{1}{15}$ cm/sec

Ans

28. If $f(x) = \sin(2x) + \ln(x+1)$, then $f'(0) =$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Ans

EXAM VI
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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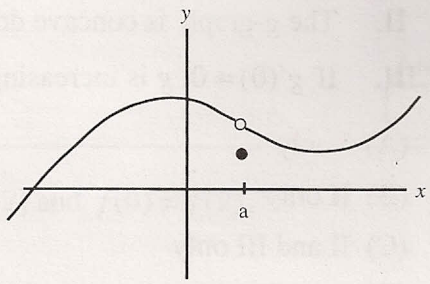
1. The graph of a function f is shown to the right. Which of the following statements about f is false?



(A) f has a relative minimum at $x = a$.

(B) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

(C) $\lim_{x\to a} f(x) \neq f(a)$

(D) $f(a) > 0$

(E) $f'(a) < 0$

Ans

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2. The function f defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

(A) -0.144 (B) -0.150 (C) -0.156 (D) -0.162 (E) -0.168

Ans

3. Boyle's Law states that if the temperature of a gas remains constant, then the pressure P and the volume V of the gas satisfy the equation $PV = c$ where c is a constant. If the volume is decreasing at the rate of 10 in^3 per second, how fast is the pressure increasing when the pressure is 100 lb/in^2 and the volume is 20 in^3?

(A) $5\,\frac{\text{lb/in}^2}{\text{sec}}$ (B) $10\,\frac{\text{lb/in}^2}{\text{sec}}$ (C) $50\,\frac{\text{lb/in}^2}{\text{sec}}$ (D) $200\,\frac{\text{lb/in}^2}{\text{sec}}$ (E) $500\,\frac{\text{lb/in}^2}{\text{sec}}$

Ans

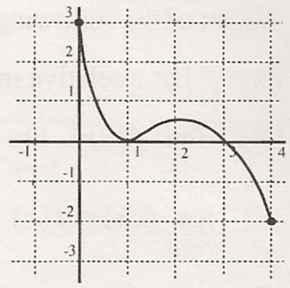
4. The graph of the second derivative of a function g is shown in the figure. Use the graph to determine which of the following are true.

I. The g-graph has points of inflection at $x = 1$ and $x = 3$.

II. The g-graph is concave down on the interval (3, 4).

III. If $g'(0) = 0$, g is increasing at $x = 2$.

Graph of g''



(A) I only
(B) II only
(C) II and III only
(D) I and II only
(E) I, II, III

Ans

5. A particle moves along a straight line with its position at any time $t \geq 0$ given by

$s(t) = \int_0^t (x^3 - 2x^2 + x)\, dx$, where s is measured in meters and t in seconds. The maximum velocity attained by the particle on the interval $0 \leq t \leq 3$ is

(A) 0.333 m/sec
(B) 0.148 m/sec
(C) 1 m/sec
(D) 3 m/sec
(E) 12 m/sec

Ans

6. If $\frac{dy}{dx} = \sqrt{2x+1}$, then the average rate of change y with respect to x on the closed interval $[0, 4]$ is

(A) 13 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) $\frac{13}{6}$ (E) $\frac{1}{9}$

Ans

7. If f' is a continuous function on the closed interval $[0, 2]$ and $f(0) = f(2)$, then $\int_0^2 f'(x)\, dx =$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Ans

8. If $k \neq 0$, then $\lim_{x \to k} \frac{x^2 - k^2}{x^2 - kx} =$

(A) 0

(B) 2

(C) $2k$

(D) $4k$

(E) nonexistent

Ans

9. Suppose that, during the first year after its hatching, the weight of a duck increases at a rate proportional to its weight. The duckling weighed 2 pounds when it was hatched and 3.5 pounds at age 4 months. How many pounds will the bird weigh at age 6 months?

(A) 4.2 lbs

(B) 4.6 lbs

(C) 4.8 lbs

(D) 5.6 lbs

(E) 6.5 lbs

Ans

10. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y = \ln x$ from $x = 1$ to $x = 4$. If the Trapezoid Rule with 3 subdivisions is used to approximate the area of R, the approximation is

(A) 4.970 (B) 2.510 (C) 2.497 (D) 2.485 (E) 2.473

Ans

11. A solid has as its base the region enclosed by the graph of $y = \cos x$ and the x-axis between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. If every cross section perpendicular to the x-axis is a square, the volume of the solid is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi^2}{2}$

(E) 2

Ans

12. If the function f is differentiable at the point $(a, f(a))$, then which of the following are true?

I. $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

II. $f'(a)=\lim_{h\to 0}\frac{f(a)-f(a-h)}{h}$

III. $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, III

Ans

13. The level of air pollution at a distance x miles from a tire factory is given by

$$L(x)=e^{-0.1x}+\frac{1}{x^2}.$$

The average level of pollution between 15 and 25 miles from the factory is

(A) 0.144
(B) 0.250
(C) 0.156
(D) 0.162
(E) 0.168

Ans

14. Suppose the continuous function f is defined on the closed interval [0, 3] such that its derivative f' is defined by $f'(x) = e^x \sin(x^2) - 1$. Which of the following are true about the graph of f?

I. f has exactly one relative maximum point.
II. f has two relative minimum points.
III. f has two inflection points.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Ans

15. If the average value of $y = x^2$ over the interval $[1, b]$ is $\frac{13}{3}$, then the value of b could be

(A) $\frac{7}{3}$ (B) 3 (C) $\frac{11}{3}$ (D) 4 (E) $\frac{13}{3}$

Ans

16. If the function f is defined on the closed interval [0, 3] by $f(x) = \dfrac{2x}{x^2+1}$, which of the following is true?

I. $\int_0^3 f(x)\,dx = \ln 10$

II. f has a relative maximum at $x = 1$.

III. $f'(2) = \frac{1}{2}$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

Ans $\square$

17. The area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is approximately

(A) 10.80
(B) 10.97
(C) 11.14
(D) 11.31
(E) 11.48

Ans $\square$

EXAM VI
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of problems–3

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let R be the region in the first quadrant under the graph of $y = \dfrac{8}{\sqrt[3]{x}}$ for $1 \le x \le 8$.

(a) Find the area of R.

(b) The line $x = k$ divides the region R into two regions. If the part of region R to the left of the line is $\frac{5}{12}$ of the area of the whole region R, what is the value of k ?

(c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are semicircles.

2. A particle starts at the point $(1,0)$ at $t = 0$ and moves along the x-axis so that at time $t \geq 0$ its velocity $v(t)$ is given by $v(t) = 1 + \dfrac{t}{1+t^2}$.

(a) Determine the maximum velocity of the particle. Show your work.

(b) Find an expression for the position $s(t)$ of the particle at time t.

(c) What is the limiting value of the velocity as t increases without bound?

(d) Determine for which values of t, if any, the particle reaches the point $(101,0)$.

3. The rate at which an air-conditioning unit for a theater complex pumps out cool air, in metric tons per hour, is given by a differentiable function R of time t. The table shows the rate as measured every hour over an 8-hour time period.

t (hours)	$R(t)$ (metric tons per hour)
0	4.6
1	5.4
2	6.1
3	6.5
4	6.8
5	6.3
6	6.1
7	5.5
8	4.8

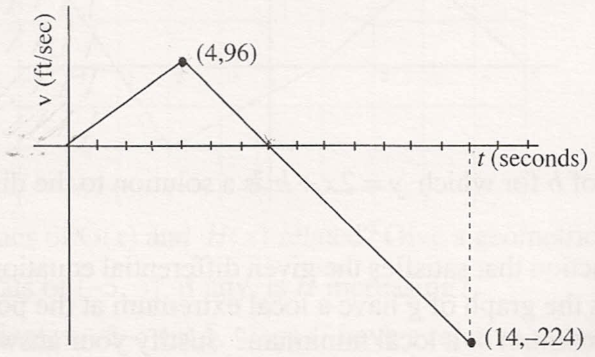
(a) Use a midpoint Riemann sum with 4 subintervals of equal length to approximate $\int_0^8 R(t)\,dt$. Explain, using correct units, the meaning of your answer in terms of air flow.

(b) Is there some time t, $0 < t < 8$, such that $R'(t) = 0$? Explain.

(c) The rate of air flow $R(t)$ can be approximated using $Q(t) = \frac{1}{8}\left(36 + 8t - t^2\right)$. Use $Q(t)$ to approximate the average rate of air flow during the 8-hour time period.

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

4. The graph below shows the velocity v in feet per second of a small rocket that was fired from the top of a tower at time $t = 0$ (t in seconds). The rocket accelerated with constant upward acceleration until its fuel was expended, then fell back to the ground at the foot of the tower. The entire flight lasted 14 seconds.



(a) What was the acceleration of the rocket while its fueled lasted?

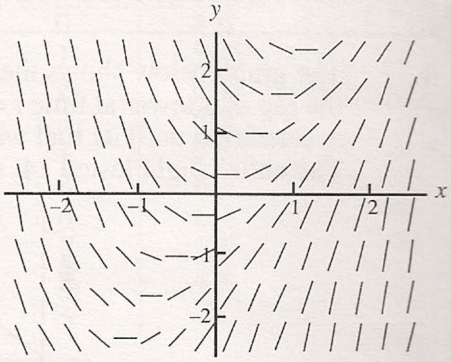
(b) How long was the rocket rising?

(c) What was the maximum height above the ground that the rocket reached?

(d) How high was the tower from which the rocket was fired?

5. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(1, 0)$ and sketch the solution curve that passes through the point $(0, 1)$.



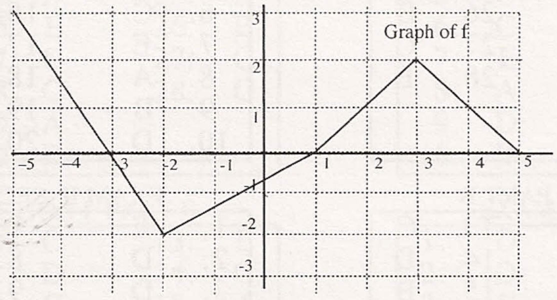
(b) Find the value of b for which $y = 2x + b$ is a solution to the differential equation. Justify your answer.

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

(d) Show that if C is a constant, then $y = Ce^{-x} + 2x - 2$ is a solution of the differential equation

6. A differentiable function f is defined on the closed interval $[-5, 5]$ and a graph of f is shown in the figure below. Functions G and H are defined on the interval $[-5, 5]$ by

$$G(x) = \int_{-3}^{x} f(t)\,dt \text{ and } H(x) = \int_{2}^{x} f(t)\,dt\ .$$



(a) How are the values of G(x) and $H(x)$ related? Give a geometric explanation of this relationship.

(b) On which intervals of $[-5, 5]$, if any, is H increasing?

(c) At what x-coordinates, $-5 < x < 5$, does G have a relative maximum? Justify your answer.

(d) On which subintervals of $[-5, 5]$, if any, is G concave up?